WJHS Summer Math Packet
For Rising AP Calculus AB and Calculus with Applications Students

This packet is an optional review of the skills that will help you be successful in AP Calculus AB and Calculus with Applications. By completing this packet over the summer, you will not only keep your brain mathematically active but you will be able to identify skills that you need to strengthen for your year ahead. Complete the exercises in the space provided then check your answers with the Answer Key. If you struggle with any of the exercises, please seek help from a friend, parent, sibling, book, or online resource. **Unless specified, you should _NOT_ use a calculator**. Enjoy your math review and we look forward to meeting you in August!

I. Factor each of the following expressions.

1. $3x^2 + 6x$

$$3x(x+2)$$

2. $x^2 + 5x + 6$

$$(x+2)(x+3)$$

3. $3x^2 - x - 2$

$$(3x+2)(x-1)$$

4. $x^2 - 8x + 15$

$$(x-3)(x-5)$$

5. $9x^2 - 9x + 2$

$$(3x-1)(3x-2)$$

6. $x^2 - 14x + 45$

$$(x-9)(x-5)$$

7. $x^2 - 10x + 25$

$$(x-5)^2$$

8. $64x^2 - 49y^2$

$$(8x-7y)(8x+7y)$$

9. $3x^3 - 2x^2 - 6x + 4$

$$(x^2-2)(3x-2)$$

II. Simplify each of the following expressions.

1. $\dfrac{x-4}{x^2-3x-4} = \dfrac{1}{x+1}$

2. $\dfrac{x^3-8}{x-2} = x^2 + 2x + 4$

3. $\dfrac{5-x}{x^2-25} = \dfrac{-1}{x-5} = \dfrac{1}{5-x}$

4. $\dfrac{x^2-4x-32}{x^2-16} = \dfrac{x-8}{x-4}$

5. $\dfrac{1}{x+h}-\dfrac{1}{x}=$ $\dfrac{-h}{x^2+xh}$

6. $\dfrac{\frac{2}{x^2}}{\frac{10}{x^5}}=$ $\dfrac{x^3}{5}$

7. $\dfrac{\frac{1}{3+x}-\frac{1}{3}}{x}=$ $\dfrac{-1}{9+3x}$

8. $\dfrac{2x}{x^2-6x+9}-\dfrac{1}{x+1}-\dfrac{8}{x^2-2x-3}=\dfrac{x^2+15}{(x+1)(x-3)^2}$

9. $\dfrac{\frac{25}{a}-a}{5+a}=$ $\dfrac{5-a}{a}$

10. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}=\dfrac{x^2-x-1}{x^2+x+1}$

III. Solve for z.

1. $4x + 10yz = 0$

$$z = \frac{-2x}{5y}$$

2. $y^2 + 3yz - 8z - 4x = 0$

$$z = \frac{4x - y^2}{3y - 8}$$

IV. Simplify.

1. $\dfrac{\sqrt{x}}{x^{\frac{2}{3}}}$ $= \dfrac{1}{\sqrt[6]{x}}$

2. $e^{\ln 3}$ $= 3$

3. $e^{(1+\ln x)}$ $= ex$

4. $\ln 1$ $= 0$

5. $\ln e^7$ $= 7$

6. $\log_3(1/3)$ $= -1$

7. $\log_{1/2} 8$ $= -3$

8. $\ln \dfrac{24}{\sqrt{e}}$ $= \ln 24 - \dfrac{1}{2}$

9. $e^{3\ln x}$ $= x^3$

10. $\dfrac{4xy^{-2}}{12x^{\frac{1}{3}}y^{-5}}$ $= \dfrac{x^{4/3}y^3}{3}$

11. $27^{2/3}$ $= 9$

12. $(5a^{2/3})(4a^{3/2}) = 20a^{13/6}$

V. Write the equation of a line with each of the following characteristics in point slope form.
$(y - y_1) = m(x - x_1)$

1. slope $m = -2$ and containing the point (5, 4)

$$y - 4 = -2(x - 5)$$

2. containing the two points (1,-3) and (-5, 2)

$$y - 2 = -\frac{5}{6}(x + 5)$$

3. slope $m = 0$ and containing the point (1, -6)

$$y = -6$$
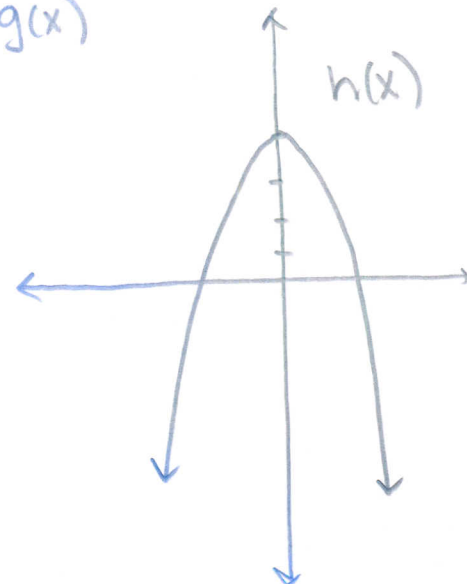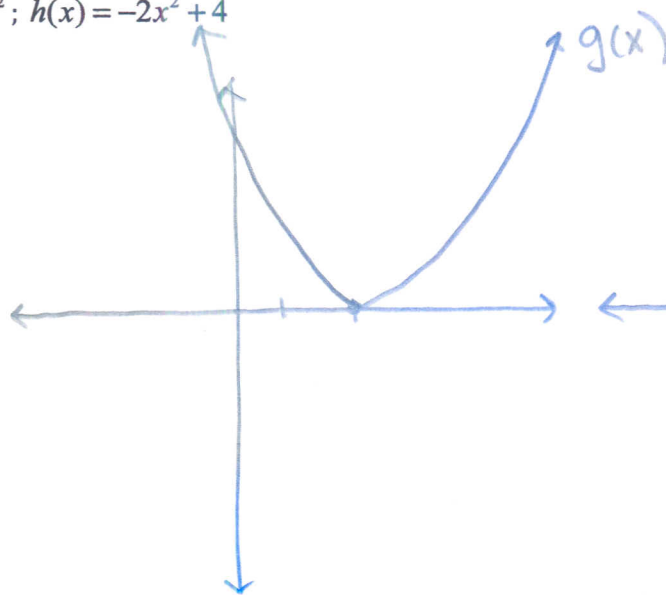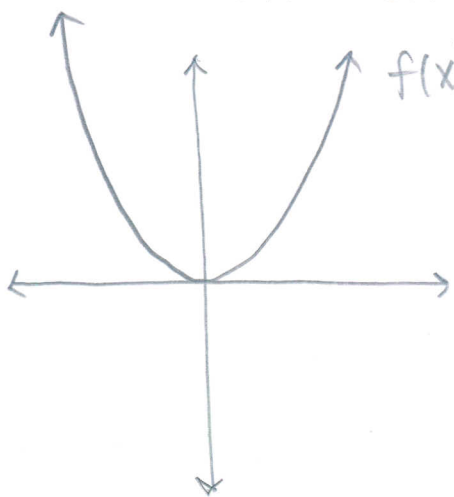
4. parallel to 2x – 3y = 7 and passing through (5 , 1)

$$y - 1 = \frac{2}{3}(x - 5)$$

5. perpendicular to the line in problem #1 and containing the point (3, 4)
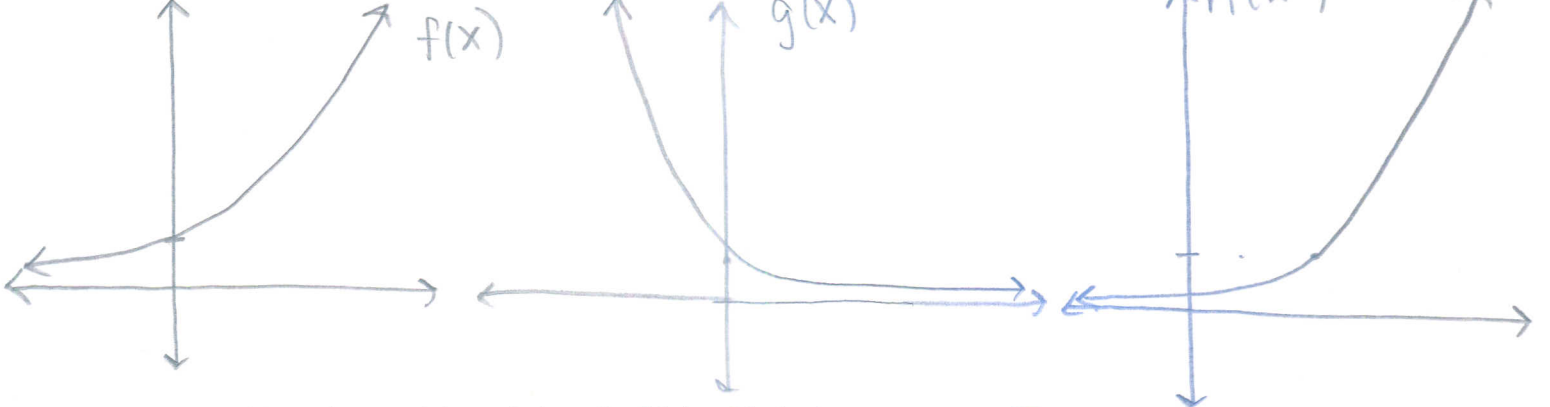
$$y - 4 = \frac{1}{2}(x - 3)$$

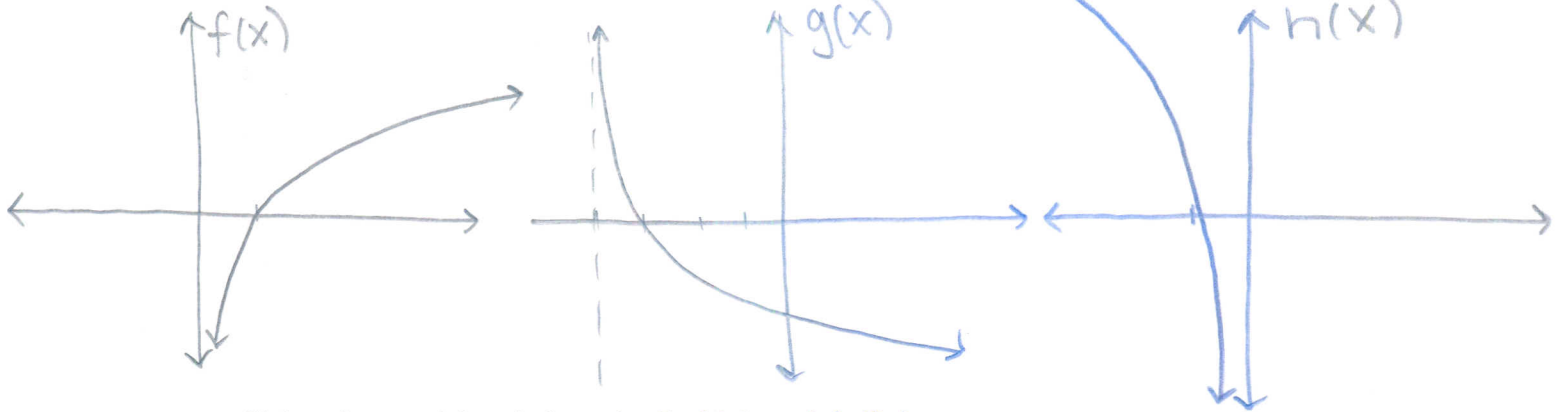VI. Without a calculator, sketch each parent function and the transformations of it.
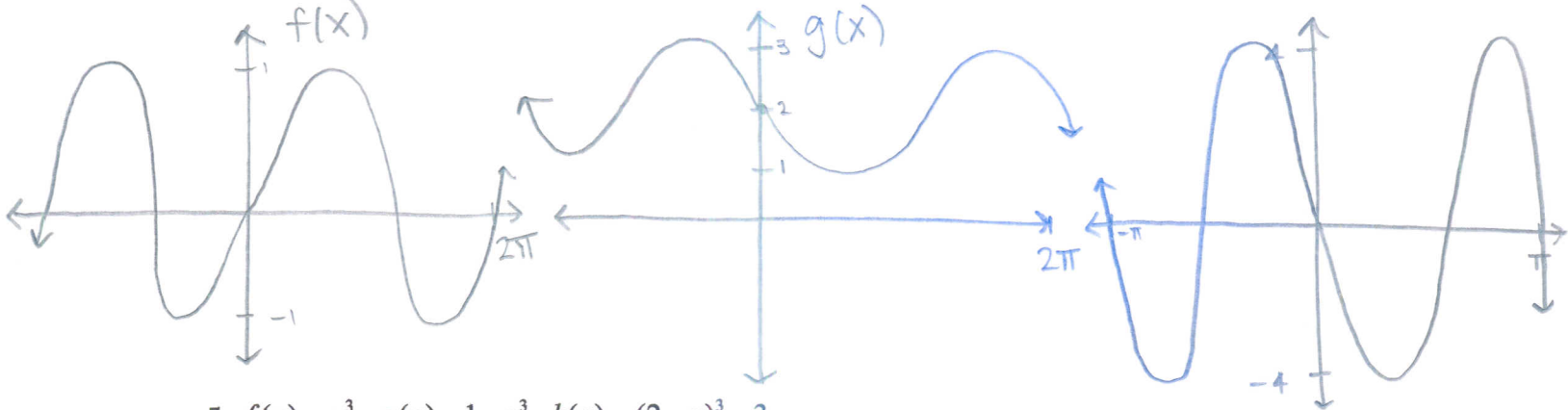
1. $f(x) = x^2$; $g(x) = (x - 2)^2$; $h(x) = -2x^2 + 4$
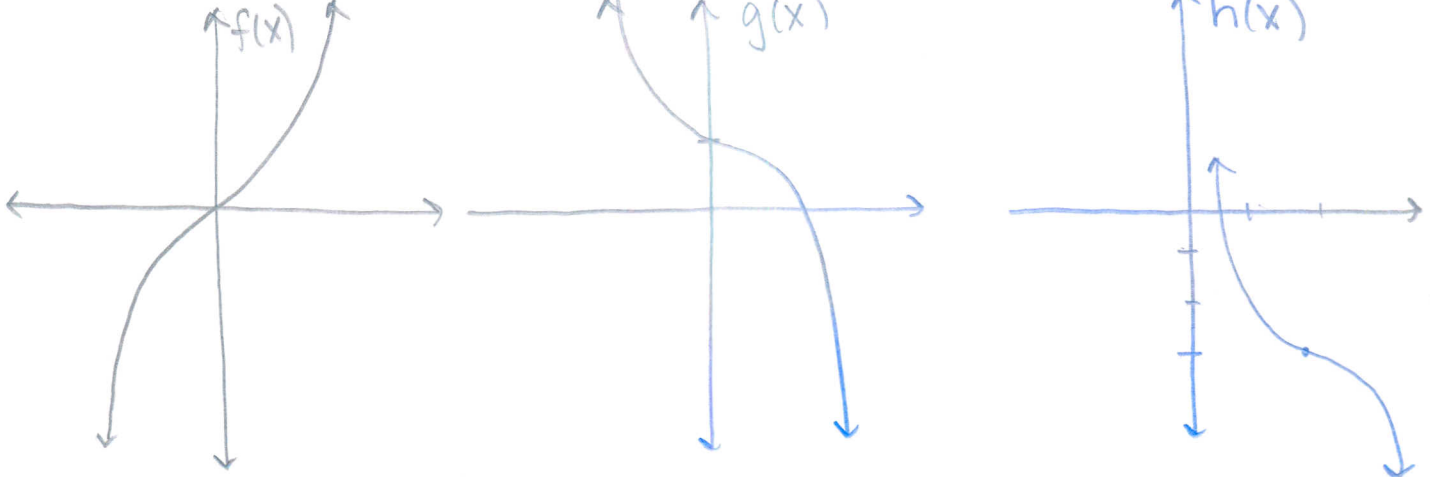
2. $f(x) = e^x$; $g(x) = e^{-x}$; $h(x) = e^{x-2}$

f(x)　　　　g(x)　　　　h(x)

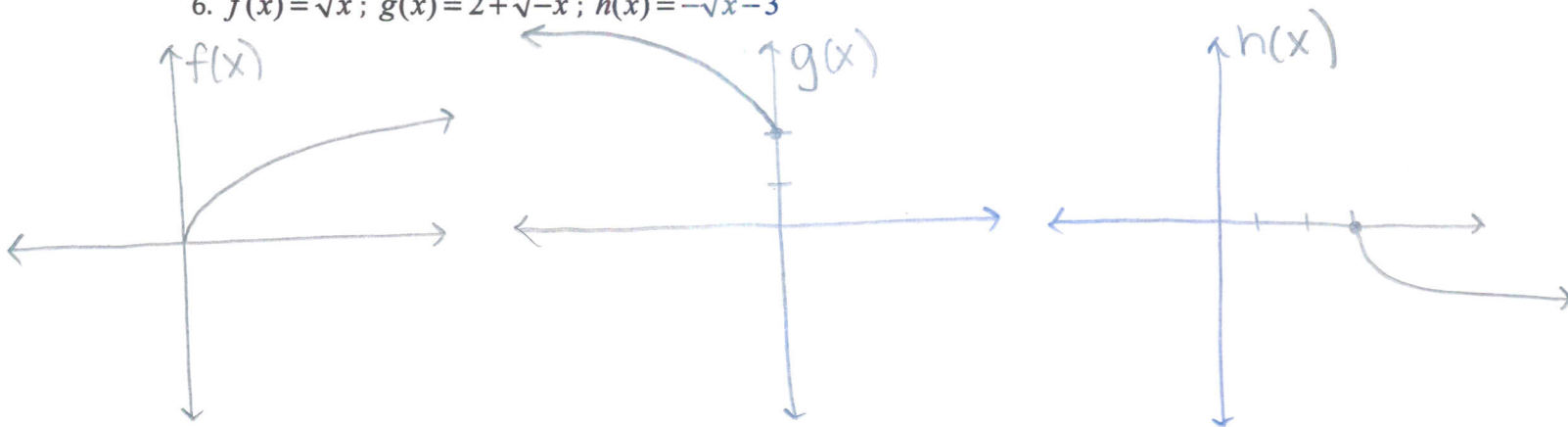3. $f(x) = \ln x$; $g(x) = -\ln(x+4)$; $h(x) = 2\ln(-x)$

f(x)　　　　g(x)　　　　h(x)

4. $f(x) = \sin x$; $g(x) = \sin(x-\pi)+2$; $h(x) = -4\sin(2x)$

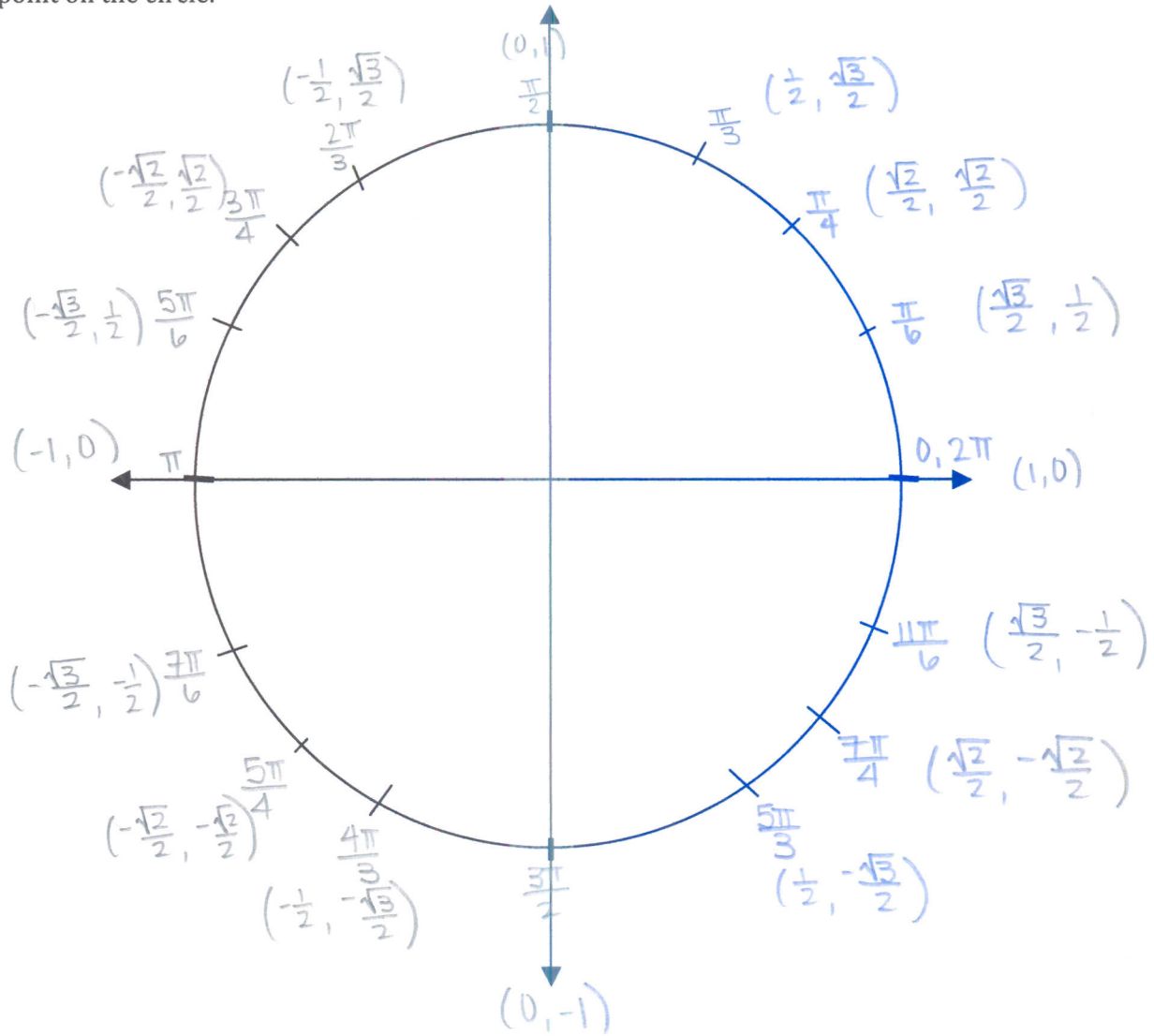f(x)　　　　g(x)　　　　h(x)

5. $f(x) = x^3$; $g(x) = 1-x^3$; $h(x) = (2-x)^3 - 3$

f(x)　　　　g(x)　　　　h(x)

6. $f(x) = \sqrt{x}$; $g(x) = 2 + \sqrt{-x}$; $h(x) = -\sqrt{x-3}$

VII. Label the unit circle with angles (in radians) and the x and y-coordinates of each important point on the circle.

VIII. Without a calculator, complete the following table for angles $0 \le x \le 360$.

Angle in Degrees (x)	Angle in Radians (x)	sin x	cos x	tan x
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	undefined
150°	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
300°	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315°	$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
330°	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
225°	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1

IX. Without a calculator, evaluate each of the following. All angle measures should be written in radians.

1. $\csc\left(-\frac{5\pi}{6}\right) = -2$

2. $\sec \pi = -1$

3. $\cot\left(-\frac{\pi}{2}\right) = 0$

4. $\tan\frac{\pi}{2} = $ undefined

5. $\sin\frac{5\pi}{6} = \frac{1}{2}$

6. $\cot\frac{2\pi}{3} = -\frac{\sqrt{3}}{3}$

7. $\sin\frac{\pi}{2} = 1$

8. $\sec\left(-\frac{3\pi}{4}\right) = -\sqrt{2}$

9. $\csc \pi = $ undefined

10. $\sec\frac{11\pi}{6} = \frac{2\sqrt{3}}{3}$

11. $\cot\frac{4\pi}{3} = \frac{\sqrt{3}}{3}$

12. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

X. Without a calculator, identify all asymptotes and points of discontinuity in each of the following functions.

1. $f(x) = \dfrac{5x^2 + x}{5x + 1}$

hole at $\left(-\frac{1}{5}, -\frac{1}{5}\right)$

2. $g(x) = \dfrac{3x^2 + x - 9}{2x - 4}$

v.a $x = 2$
h.a. none
s.a. $y = \frac{3}{2}x + \frac{7}{2}$

3. $h(x) = \dfrac{2x - 5}{7x + 1} + 3$

v.a. $x = -\frac{1}{7}$
h.a. $y = \frac{23}{7}$

4. $j(x) = \dfrac{2}{9x - 18}$

v.a $x = 2$
h.a. $y = 0$

XI. Without a calculator, identify the end behavior of each of the following functions.

1. $f(x) = -x^4 + 3x^3 + 11x - 1$

as $x \to -\infty$, $f(x) \to -\infty$
as $x \to \infty$, $f(x) \to -\infty$

2. $g(x) = 3x^5 + 2x^2$

as $x \to -\infty$, $g(x) \to -\infty$
as $x \to \infty$, $g(x) \to \infty$

3. $h(x) = 2\log_2(x - 3)$

as $x \to 3^+$, $h(x) \to -\infty$
as $x \to \infty$, $h(x) \to \infty$
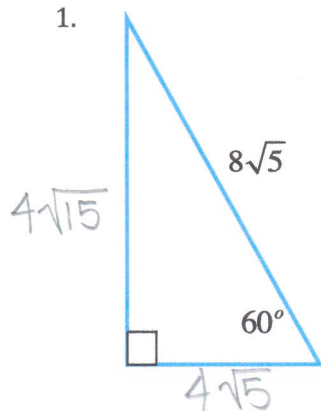
4. $j(x) = \dfrac{10x - 4}{4x + 1}$
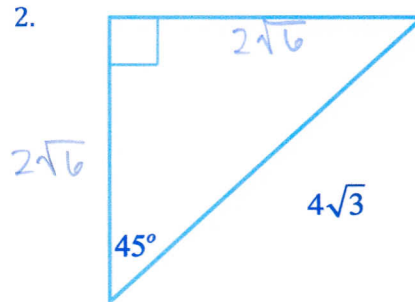
as $x \to -\infty$, $j(x) \to \frac{5}{2}$
as $x \to \infty$, $j(x) \to \frac{5}{2}$

XII. Solve for each of the missing sides.

1.

$8\sqrt{5}$

$4\sqrt{15}$

$60°$

$4\sqrt{5}$

2.

$2\sqrt{6}$

$2\sqrt{6}$

$4\sqrt{3}$

$45°$

XIII. Given the functions f, g, h, evaluate/simplify each of the following.

$$f(x) = x^2 + x \qquad g(x) = \sqrt{x} \qquad h(x) = \frac{1}{x}$$
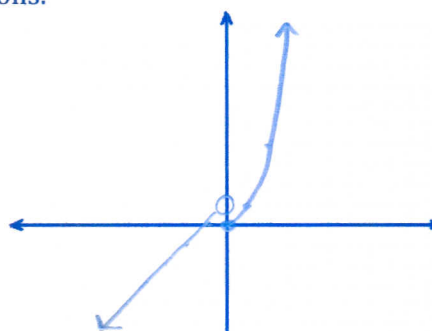
1. $g(3) = \sqrt{3}$

2. $g(h(9)) = \frac{1}{3}$

3. $h(x+4) = \frac{1}{x+4}, \quad x \neq -4$

4. $f(x+t) = x^2 + 2xt + t^2 + x + t$

XIV. Graph each of the following piecewise functions.

1. $f(x) = \begin{cases} x+1, x < 0 \\ x^2, x \geq 0 \end{cases}$

2. $j(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, x \neq 2 \\ 5, x = 2 \end{cases}$