AP AB Calculus

Please note: The official College Board® Course Description is available below in "More."

Calculus AB is a course in single-variable calculus that includes techniques and applications of the derivative, techniques and applications of the definite integral, and the Fundamental Theorem of Calculus. It is equivalent to at least a semester of calculus at most colleges and universities, perhaps to a year of calculus at some. Algebraic, numerical, and graphical representations are emphasized throughout the course.

So that students might better appreciate the connections among various representations, it is assumed that they will have access to graphing calculators in class and on homework. To that end, and to shift the emphasis from mere computation to a deeper understanding of concepts, graphing calculators are required on portions of the AP Examination.

Prerequisites for the student include two years of algebra and a year of geometry, plus a strong grounding in elementary functions and their graphs, including trigonometry (usually gained in an additional course called precalculus or college algebra). Attitude prerequisites include a willingness to work both in and out of class, a willingness to collaborate with classmates to foster mutual understanding, and a sincere intent to place out of the first semester of college calculus rather than repeat it.

Prerequisites for the teacher include a good understanding of calculus, a willingness to teach and learn from good students, and (if possible) the ability to attend a College Board® AP workshop or Summer Institute to communicate the goals of the course.

Although the goal for students should be to learn calculus rather than to succeed on a single assessment, the desire to do well on the AP Examination can be a strong motivation for students and an exploitable resource for teachers. It is recommended that teachers obtain copies of old AP Examinations (from colleagues or from the College Board), learn how they are scored, and hold students to that level of performance throughout the course.

Goals

· Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal.
They should understand the connections
among these representations.
· Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation, and should be able to use derivatives to solve a variety of problems.
· Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change, and should be able to use integrals to solve a variety of problems.
· Students should understand the relationship between the derivativeand
the definite
integral as expressed in both parts of the Fundamental Theorem of Calculus.
· Students should be able to communicate mathematics and explain solutions to problems both verbally and in written sentences.
· Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.
· Students should be able to use technology to help solve problems, experiment, interpret results, and support conclusions.

· Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
· Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Prerequisites

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. In particular, before studyingcalculus,
students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry,
zeros, intercepts,
and so on) and know the values of the trigonometric functions at the numbers
0, _6 , _4 , _3 , _2 ,, and their multiples.

Topic Outline for Calculus AB

This topic outline is intended to indicate the scope of the course, but it is not necessarily the order in which the topics need to be taught. Teachers may find that topics are best taught in different orders. (See AP Central [apcentral.collegeboard .org] for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

· An intuitive understanding of the limiting process.
· Calculating limits using algebra.
· Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior

· Understanding asymptotes in terms of graphical behavior.·
Describing asymptotic behavior in terms of limits involving infinity.
· Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth).

Continuity as a property of functions

· An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
· Understanding continuity in terms of limits.
· Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

II. Derivatives

Concept of the derivative

· Derivative presented graphically, numerically, and analytically.
· Derivative interpreted as an instantaneous rate of change.
· Derivative defined as the limit of the difference quotient.
· Relationship between differentiability and continuity.

Derivative at a point

· Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
· Tangent line to a curve at a point and local linear approximation.
· Instantaneous rate of change as the limit of average rate of change.
· Approximate rate of change from graphs and tables of values.

Derivative as a function

· Corresponding characteristics of graphs of f and f'.
· Relationship between the increasing and decreasing behavior of f and the sign of f'.
· The Mean Value Theorem and its geometric interpretation.
· Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

· Corresponding characteristics of the graphs of f, f', and f''.
· Relationship between the concavity of f and the sign of f''.
· Points of inflection as places where concavity changes.

Applications of derivatives

· Analysis of curves, including the notions of monotonicity and concavity.
· Optimization, both absolute (global) and relative (local) extrema.
· Modeling rates of change, including related rates problems.
· Use of implicit differentiation to find the derivative of an inverse function.
· Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
· Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

Computation of derivatives

· Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
· Derivative rules for sums, products, and quotients of functions.
· Chain rule and implicit differentiation.

III. Integrals

Interpretations and properties of definite integrals

· Definite integral as a limit of Riemann sums.
· Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

· Basic properties of definite integrals (examples include additivityand linearity).

Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.

Fundamental Theorem of Calculus

· Use of the Fundamental Theorem to evaluate definite integrals.

· Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation

· Antiderivatives following directly from derivatives of basic functions.

· Antiderivatives by substitution of variables (including change of limits for definite integrals).

Applications of antidifferentiation

· Finding specific antiderivatives using initial conditions, including applications to motion along a line.

· Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth).

Numerical approximations to definite integrals. Use of Riemannsums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

AP BC Calculus

A Deeper Understanding of Calculus Concepts

Calculus BC is a course in single-variable calculus that includes all the topics of Calculus AB (techniques and applications of the derivative, techniques and applications of the definite integral, and the Fundamental Theorem of Calculus) plus additional topics in differential and integral calculus (including parametric, polar, and vector functions) and series. It is equivalent to at least a year of calculus at most colleges and universities. Algebraic, numerical, and graphical representations are emphasized throughout the course.

So that students might better appreciate the connections among various representations, it is assumed that they will have access to graphing calculators in class and on homework. To that end, and to shift the emphasis from mere computation to a deeper understanding of concepts, graphing calculators are required on portions of the AP Examination.

Prerequisites for the student include two years of algebra and a year of geometry, plus a strong grounding in elementary functions and their graphs, including trigonometry (usually gained in an additional course called precalculus or college algebra). Some schools precede Calculus BC with an accelerated precalculus course that covers some of the early topics in differential calculus, thereby freeing up more time for BC topics in the following year. Attitude prerequisites include a willingness to work both in and out of class, a willingness to collaborate with classmates to foster mutual understanding, and a sincere intent to place out of the first year of college calculus rather than repeat it.

Prerequisites for the teacher include a good understanding of calculus, a willingness to teach and learn from good students, and (if possible) the ability to attend a College Board® AP workshop or Summer Institute to communicate the goals of the course.

Although the goal for students should be to learn calculus rather than to succeed on a single assessment, the desire to do well on the AP Examination can be a strong motivation for students and an exploitable resource for teachers. It is recommended that teachers obtain copies of old AP Examinations (from colleagues or from the College Board), learn how they are graded, and hold students to that level of performance throughout the course.

Topic Outline for Calculus BC

The topic outline for Calculus BC includes all Calculus AB topics. Additional topics are found in paragraphs that are marked with a plus sign (+) or an asterisk (*). The additional topics can be taught anywhere in the course that the instructor wishes. Some topics will naturally fit immediately after their Calculus AB counterparts. Other topics may fit best after the completion of the Calculus AB topic outline. (See AP Central for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

· An intuitive understanding of the limiting process.

· Calculating limits using algebra.

· Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior

· Understanding asymptotes in terms of graphical behavior.

· Describing asymptotic behavior in terms of limits involving infinity.

· Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth).

Continuity as a property of functions

· An intuitive understanding of continuity. (The function values can be made as

close as desired by taking sufficiently close values of the domain.)
· Understanding continuity in terms of limits.

· Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

*** Parametric, polar, and vector functions.** The analysis of planar curves includes those given in parametric form, polar form, and vector form.

II. Derivatives

Concept of the derivative

· Derivative presented graphically, numerically, and analytically.
· Derivative interpreted as an instantaneous rate of change.
· Derivative defined as the limit of the difference quotient.
· Relationship between differentiability and continuity.

Derivative at a point

· Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
· Tangent line to a curve at a point and local linear approximation.
· Instantaneous rate of change as the limit of average rate of change.
· Approximate rate of change from graphs and tables of values.

Derivative as a function

· Corresponding characteristics of graphs of f and f'.
· Relationship between the increasing and decreasing behaviorof f and the sign of f'.
· The Mean Value Theorem and its geometric interpretation.
· Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

· Corresponding characteristics of the graphs of f, f', and f''.
· Relationship between the concavity of f and the sign of f''.
· Points of inflection as places where concavity changes.

Applications of derivatives

· Analysis of curves, including the notions of monotonicity and concavity.
+ Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration.
· Optimization, both absolute (global) and relative (local) extrema.
· Modeling rates of change, including related rates problems.
· Use of implicit differentiation to find the derivative of an inverse function.
· Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
· Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.
+ Numerical solution of differential equations using Euler's method.
+ L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series.

Computation of derivatives

· Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
· Derivative rules for sums, products, and quotients of functions.
· Chain rule and implicit differentiation.
+ Derivatives of parametric, polar, and vector functions.

III. Integrals

Interpretations and properties of definite integrals

· Definite integral as a limit of Riemann sums.
· Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

· Basic properties of definite integrals (examples include additivityand linearity).

*** Applications of integrals.** Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from a rate of change.

Fundamental Theorem of Calculus

· Use of the Fundamental Theorem to evaluate definite integrals.
· Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation

· Antiderivatives following directly from derivatives of basic functions.
+ Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only).
+ Improper integrals (as limits of definite integrals).

Applications of antidifferentiation

· Finding specific antiderivatives using initial conditions, including applications to motion along a line.
· Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth).

+ Solving logistic differential equations and using them in modeling.

Numerical approximations to definite integrals. Use of Riemannsums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

***IV. Polynomial Approximations and Series**

*** Concept of series.** A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequenceof partial sums.

Technology can be used to explore convergence and divergence.

*** Series of constants**

+ Motivating examples, including decimal expansion.
+ Geometric series with applications.
+ The harmonic series.
+ Alternating series with error bound.
+ Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series.
+ The ratio test for convergence and divergence.
+ Comparing series to test for convergence or divergence.

*** Taylor series**

+ Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximatingthe sine curve).
+ Maclaurin series and the general Taylor series centered at $x = a$.
+ Maclaurin series for the functions e^x, sin x, cos x,, and $\frac{1}{1-x}$.
+ Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.
+ Functions defined by power series.
+ Radius and interval of convergence of power series.
+ Lagrange error bound for Taylor polynomials.

AP Statistics

Unlike the AP Statistics course description, which outlines the scope and nature of the course, this introduction focuses on the teaching and learning experience. The teachers who pioneered this program were deeply committed to it and excited about the benefits it could bring to their students. I believe their enthusiasm was rooted in (1) the discipline of statistics, (2) their experience with their students, and (3) the collective professional community they created.

Because the science, art, and practice of statistics differ significantly from other fields of mathematics, it is not surprising that this discipline is also taught differently. Among ecologists, there is a concept known as the "edge effect," the biologically active, interstitial region that forms a boundary --, for example, between a forest and a meadow. As the eminent statistician John Tukey noted, the field of statistics allows you to play in everyone else's backyard. Statistics is positioned at the edge between the known and the unknown in all those backyards. Our classes are populated with students who possess a bewildering variety of interests, some of which are allegedly nonmathematical. Statistics can encompass and expand those interests, and provide the methods and concepts for creatively extending knowledge in all of their backyards.

The AP Statistics classroom is nothing if not active. Students analyze data with calculators and computers, conduct classroom experiments, carry out individual and group projects, and perform simulations involving probabilistic concepts. AP Statistics students are active, engaged learners. Moreover, these students would not necessarily be enchanted by a traditional mathematics course. The AP Statistics course not only accommodates students with a wide variety of interests, it also serves those with a relatively wide range of math proficiency. Discussion in an AP Statistics class is an activity for all students. Group projects are less likely to be dominated by the most able student, and individuals can succeed by capitalizing on their individual interests. A more healthy learning and teaching environment is difficult to imagine.

It has long been a fact of life that AP Statistics teachers are lonely members of their math departments. The preservice preparation of most math teachers today does not include a statistics course, and high school statistics teachers have less opportunity to bounce ideas off their colleagues. (This phenomenon is also not unknown among statisticians teaching in some colleges.) Ironically, this isolation, together with the power of the Internet, has spawned what is possibly the most collegial resource available to high school teachers -- each other. From the very beginning of AP Statistics, an electronic discussion group (EDG) operating out of British Columbia attracted statistics teachers to ongoing discussions of content, philosophy, and pedagogy. This EDG, now operating under the aegis of the College Board®, created a synergistic network that has aided hundreds of high school teachers, as well as college and university professors who realize the importance of the precollege statistics curriculum. Sessions at the annual meetings of the National Council of Teachers of Mathematics, statistics institutes at the North Carolina School of Science and Mathematics, and a growing number of Web sites have been direct consequences of this long-range collegiality. Clearly, this is the way our profession ought to work -- and nobody has done it better than the AP Statistics teachers.

In the light of the first six years' experience, the AP Statistics phenomenon must be declared an incredible and enduring success. It is a success not merely by the numbers -- 173,944 students have taken the AP Statistics exam in the past six years -- but because of the personal and professional experience of teachers like you, and the learning experience of students like yours.

Topic Outline

Following is an outline of the major topics covered by the AP Statistics Exam. The ordering here is intended to define the scope of the course but not necessarily the sequence. The percentages in parentheses for each content area indicate the coverage for that content area in the exam.

I. Exploring Data: Describing patterns and departures from patterns (20%-30%)

Exploratory analysis of data makes use of graphical and numerical techniques to study patterns and departures from patterns. Emphasis should be placed on interpreting information from graphical and numerical displays and summaries.

A. _Constructing and interpreting graphical displays of distributions of univariate data (dotplot, stemplot, histogram, cumulative frequency plot)

1. Center and spread
2. Clusters and gaps

3.	Outliers and other unusual features
4.	Shape
B.	Summarizing distributions of univariate data
1.	Measuring center: median, mean
2.	_Measuring spread: range, interquartile range, standard deviation
3.	_Measuring position: quartiles, percentiles, standardized scores (z-scores)
4.	Using boxplots
5.	The effect of changing units on summary measures

C.	_Comparing distributions of univariate data (dotplots, back-to-back stemplots, parallel boxplots)
1.	_Comparing center and spread: within group, between group variation
2.	Comparing clusters and gaps
3.	Comparing outliers and other unusual features
4.	Comparing shapes
D.	Exploring bivariate data
1.	Analyzing patterns in scatterplots
2.	Correlation and linearity
3.	Least-squares regression line
4.	Residual plots, outliers and influential points
5.	_Transformations to achieve linearity: logarithmic and power transformations
E.	Exploring categorical data
1.	Frequency tables and bar charts
2.	Marginal and joint frequencies for two-way tables
3.	Conditional relative frequencies and association
4. Comparing distributions using bar charts
II.	Sampling and Experimentation: Planning and conducting a study (10%-15%)
Data must be collected according to a well-developed plan if valid information on a conjecture is to be obtained. This plan includes clarifying the question and deciding upon a method of data collection and analysis.
A.	Overview of methods of data collection
1.	Census
2.	Sample survey
3.	Experiment
4.	Observational study
B.	Planning and conducting surveys
1.	_Characteristics of a well-designed and well-conducted survey
2.	Populations, samples and random selection
3.	Sources of bias in sampling and surveys
4.	_Sampling methods, including simple random sampling, stratified random sampling and cluster sampling
C.	Planning and conducting experiments
1.	_Characteristics of a well-designed and well-conducted experiment
2.	_Treatments, control groups, experimental units, random assignments and replication
3.	_Sources of bias and confounding, including placebo effect and blinding
4.	Completely randomized design

5.	Randomized block design, including matched pairs design
D.	Generalizability of results and types of conclusions that can be drawn from observational studies, experiments and surveys

III.	_Anticipating Patterns: Exploring random phenomena using probability and simulation (20%-30%)
Probability is the tool used for anticipating what the distribution of data should look like under a given model.
A.	Probability
1.	_Interpreting probability, including long-run relative frequency interpretation
2.	"Law of Large Numbers" concept
3.	_Addition rule, multiplication rule, conditional probability and independence
4.	_Discrete random variables and their probability distributions, including binomial and geometric
5.	Simulation of random behavior and probability distributions
6.	_Mean (expected value) and standard deviation of a random variable, and linear transformation of a random variable
B.	Combining independent random variables
1.	Notion of independence versus dependence
2.	_Mean and standard deviation for sums and differences of independent random variables
C.	The normal distribution
1.	Properties of the normal distribution
2.	Using tables of the normal distribution
3.	The normal distribution as a model for measurements
D.	Sampling distributions
1.	Sampling distribution of a sample proportion
2.	Sampling distribution of a sample mean
3.	Central Limit Theorem
4.	_Sampling distribution of a difference between two independent sample proportions
5.	_Sampling distribution of a difference between two independent sample means
6.	Simulation of sampling distributions
7.	t-distribution
8.	Chi-square distribution
IV.	Statistical Inference: Estimating population parameters and testing hypotheses (30%-40%)
Statistical inference guides the selection of appropriate models.
A.	Estimation (point estimators and confidence intervals)
1.	_Estimating population parameters and margins of error
2.	_Properties of point estimators, including unbiasedness and variability
3.	_Logic of confidence intervals, meaning of confidence level and confidence intervals, and properties of confidence intervals
4.	Large sample confidence interval for a proportion
5.	_Large sample confidence interval for a difference between two proportions

6. Confidence interval for a mean
7. _Confidence interval for a difference between two means (unpaired
and paired)
8. _Confidence interval for the slope of a least-squares regression line
B. Tests of significance
1. _Logic of significance testing, null and alternative hypotheses; p-values;
one- and two-sided tests; concepts of Type I and Type II errors; concept
of power
2. Large sample test for a proportion
3. Large sample test for a difference between two proportions
4. Test for a mean
5. _Test for a difference between two means (unpaired and paired)
6. _Chi-square test for goodness of fit, homogeneity of proportions, and
independence (one- and two-way tables)
7. Test for the slope of a least-squares regression line